[7]Ehlert, J. R., and Smith, T. F., "View Factors for Perpendicular and Parallel Rectangular Plates," *Journal of Thermophysics and Heat Transfer*, Vol. 7, No. 1, 1993, pp. 173–175.

[8]Kahaner, D., Moler, C., and Nash, S., *Numerical Methods and Software*, Prentice–Hall, Englewood Cliffs, NJ, 1989.

Total Emissivities of Temperature-Fluctuating Molecular Gases

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Introduction

THE problem of radiative heat transfer in a layer of hot molecular gases has been a long-standing topic in thermal engineering and during the past several decades numerous studies have been devoted to this subject. Almost all of the earlier studies, however, were concerned with radiative transfer in stationary isothermal or nonisothermal layers of infrared gases, and less attention has been paid to radiative transfer in temperature and concentration fluctuating media, which is of particular importance in relation to quantitative predictions of the amount of radiant energy from actual flames or fires. Cox[1] originally considered theoretical effects of temperature fluctuating flames on radiative heat transfer utilizing a gray gas model and concluded that a turbulent flame behaves as if it was a laminar flame of high mean temperature, but the quantitative evaluation of these effects was not made. Since Cox's work,[1] experimental and theoretical studies of radiative transports from turbulent flames have been performed by several researchers,[2–4] and it was shown that temperature fluctuations can increase spectral emerging intensities of radiation up to almost two times higher than predictions based on mean scalar fields.[2] Despite these studies, it has not yet been quantitatively understood how the fundamental system parameters, such as the temperature variance, total and partial gas pressures, path length, and mean temperature, affect the total radiant intensity emerging from temperature-fluctuating infrared gases.

The purpose of the present study is to remedy this deficiency. To this end, the total emissivities of carbon dioxide, water vapor, or their mixture are quantitatively evaluated by varying the previously mentioned parameters.

Analysis

The following assumptions are introduced:

1) Temperature at any location in a molecular gas layer of thickness L (m) is a random function of time, and this instantaneous temperature T (K) may be written in the form

$$T = \bar{T} + T' \tag{1}$$

where $\bar{T}$ represents the time-averaged value of temperature and T' is the fluctuating component ($\overline{T'} = 0$).

2) $\bar{T}$ and $\overline{T'^2}$ are constant throughout the layer.

3) The instantaneous blackbody intensity $I_{b\nu}(T)$ is expanded into the Taylor series around $\bar{T}$[4,5]:

$$I_{b\nu}(T) = I_{b\nu}(\bar{T}) + T' \left.\frac{\partial I_{b\nu}}{\partial T}\right|_{T=\bar{T}} + \frac{1}{2} T'^2 \left.\frac{\partial^2 I_{b\nu}}{\partial T^2}\right|_{T=\bar{T}} \cdots \tag{2}$$

4) The fluctuation of the absorption coefficient caused by temperature fluctuation is neglected in comparison with that of the blackbody intensity.[5]

5) Concentration fluctuation is disregarded.

Under these assumptions, the instantaneous radiant intensity normally emerging from a plane–parallel, homogeneous layer of molecular gases bounded by a vacuum is written in the form

$$I_{e\nu} = \int_0^L \kappa_\nu(y') I_{b\nu}(y') \exp\left[-\int_0^{y'} \kappa_\nu(y'')\, dy''\right] dy'$$
$$= \kappa_\nu(\bar{T}) \int_0^L I_{b\nu}[T(y')] \exp[-\kappa_\nu(\bar{T}) y']\, dy' \tag{3}$$

where κ_ν denotes the spectral absorption coefficient (m^{-1}). Time-averaging Eq. (3) yields

$$\bar{I}_{e\nu} = \left[I_{b\nu}(\bar{T}) + \frac{1}{2!} \overline{T'^2} \left.\frac{\partial^2 I_{b\nu}}{\partial T^2}\right|_{T=\bar{T}} + \cdots\right] [1 - \exp(-\kappa_\nu(\bar{T})L)] \tag{4}$$

The total emerging intensity from the layer is given by

$$\bar{I}_e = \int_0^\infty \bar{I}_{e\nu}\, d\nu \tag{5}$$

With these quantities, the total normal emissivity of a temperature-fluctuating gas layer ε_T, which is hereafter called the total emissivity, for abbreviation, is defined as follows:

$$\varepsilon_T = \bar{I}_e \Big/ \left(\frac{\sigma \bar{T}^4}{\pi}\right)$$
$$= \int_0^\infty \left[\Gamma_\nu^{(1)}(\bar{T}) + 6\left(\frac{\overline{T'^2}}{\bar{T}^2}\right)\Gamma_\nu^{(3)}(\bar{T}) + \left(\frac{\overline{T'^4}}{\bar{T}^4}\right)\Gamma_\nu^{(5)}(\bar{T}) + \cdots\right]$$
$$\times [1 - \exp(-\tau_{0\nu})]\, d\nu \tag{6}$$

where

$$\tau_{0\nu} = \kappa_\nu L, \qquad \Gamma_\nu^{(1)} = I_{b\nu}(\bar{T}) \Big/ \left(\frac{\sigma \bar{T}^4}{\pi}\right)$$

$$\Gamma_\nu^{(3)} = \left.\frac{\partial^2 I_{b\nu}}{\partial T^2}\right|_{T=\bar{T}} \Big/ \left(\frac{12\sigma \bar{T}^2}{\pi}\right), \qquad \Gamma_\nu^{(5)} = \left.\frac{\partial^4 I_{b\nu}}{\partial T^4}\right|_{T=\bar{T}} \Big/ \left(\frac{24\sigma}{\pi}\right)$$

From this definition, the quantity ε_T can be greater than unity. If the temperature fluctuation T' obeys a Gaussian distribution[6] and the Taylor series expansion of the blackbody intensity is summed up to the term of $\overline{T'^4}$, then $\overline{T'^4} = 3(\overline{T'^2})^2$ and the following expression results:

$$\varepsilon_T = \int_0^\infty [\Gamma_\nu^{(1)}(\bar{T}) + 6\alpha^2\Gamma_\nu^{(3)}(\bar{T}) + 3\alpha^4\Gamma_\nu^{(5)}(\bar{T})][1 - \exp(-\tau_{0\nu})]\, d\nu \tag{7}$$

where α represents a measure of the intensity of the fluctuating component of temperature and is defined by $\sqrt{\overline{T'^2}}/\bar{T}$.

For $\alpha \leq 0.3$, the relative contribution of the third term of the right-hand side (RHS) of Eq. (7) to ε_T is less than 1.6%, and thus, Eq. (7) is accurately approximated by

$$\varepsilon_T = \varepsilon_s^{(1)}(\bar{T}) + 6\alpha^2\varepsilon_s^{(2)}(\bar{T}) \tag{8}$$

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where

$$\varepsilon_s^{(1)}(\bar{T}) = \int_0^\infty \Gamma_\nu^{(1)}(\bar{T})[1 - \exp(-\tau_{0\nu})]\, d\nu \tag{9}$$

$$\varepsilon_s^{(2)}(\bar{T}) = \int_0^\infty \Gamma_\nu^{(3)}(\bar{T})[1 - \exp(-\tau_{0\nu})]\, d\nu \tag{10}$$

Note that $\varepsilon_s^{(1)}$ denotes the conventional total emissivity of an isothermal gas layer with temperature $\bar{T}$ and that $\varepsilon_s^{(1)}$ and $\varepsilon_s^{(2)}$ are independent of α. The second term of the RHS of Eq. (8) represents the effect of temperature fluctuation on the total emissivity. On the basis of the mass absorption coefficient k_ν (1/g m^{-2}), the optical thickness $\tau_{0\nu}$ is written as

$$\tau_{0\nu} = \kappa_\nu L = 10^5(P_aL)k_\nu/R\bar{T} \tag{11}$$

where P_aL denotes the pressure-path length product (bar m) and R is the gas constant (kJ/kg K). k_ν is evaluated utilizing the two-parameter wideband spectral model[7]:

$$k_\nu = (\zeta/K_1\omega)\exp[-\Delta\nu/K_1\omega]\tanh K_2\eta \tag{12}$$

Here, ζ is the integrated band intensity (cm^{-1}/g m^{-2}), ω is the bandwidth parameter (cm^{-1}), η is the line-overlap parameter, K_1 and K_2 are the correction parameters, and $\Delta\nu = \nu_u - \nu$ for an asymmetric band with upper limit ν_u, $\Delta\nu = \nu - \nu_l$ for an asymmetric band with lower limit ν_l, and $\Delta\nu = 2|\nu - \nu_c|$ for a symmetric band with center ν_c. ν is the wave number of radiation (cm^{-1}). The total emissivities of carbon dioxide, water vapor, or their mixture are computed from Eqs. (8–10) by varying the system parameters in the following ranges:

$$1 \le P_T \le 10 \text{ bar}, \qquad 0 \le P_a/P_T(=x) \le 1$$

$$0.001 \le P_aL \le 10 \text{ bar m}$$

$$250 \le \bar{T} \le 3000 \text{ K}, \qquad 0 \le \alpha \le 0.3$$

Here, P_T represents the total pressure. The wave-number integrals appearing in Eqs. (9) and (10) are computed numerically with the aid of a trapezoidal formula in the range of ν from 0 to 8500 cm^{-1}. A subinterval for the integration was taken to be 25 cm^{-1}. To check the accuracy of the integration, several results for the total emissivity are compared with the more accurate results obtained by a subinterval of 12.5 cm^{-1}. The comparison showed that the present results are 1.6% smaller, in the worst case, than the more accurate results. The necessary wideband model parameters are taken from Ref. 8.

Results and Discussion

Figure 1 shows the total emissivities of carbon dioxide layers at P_T = 1 (bar) and $P_a \to 0$ (bar). When the mean gas temperature is less than about 1500 K, the total emissivity increases with α over the range of pressure-path length products examined: the total emissivity for $\alpha = 0.3$ can be as high as almost twice that for $\alpha = 0$. For $\bar{T}$ greater than about 1500 K, the total emissivity is weakly affected by α, regardless of the value of P_aL.

Results for the total emissivities of water vapor are shown in Fig. 2 for $P_T = 1$ bar. As seen from this figure, the effect of α on the total emissivity diminishes with a decrease in a P_aL or with an increase in $\bar{T}$.

Figure 3 shows the total emissivities of a carbon dioxide and water vapor mixture at $P_c = P_w = 0.1013$ bar and $P_T = 1.013$ bar. Here, P_c and P_w, respectively, represent the partial pressures of carbon dioxide and water vapor. When $\bar{T}$ is less than about 1500 K, the total emissivity of the mixture increases with α, even at a small P_aL. This is because of the presence of carbon dioxide.

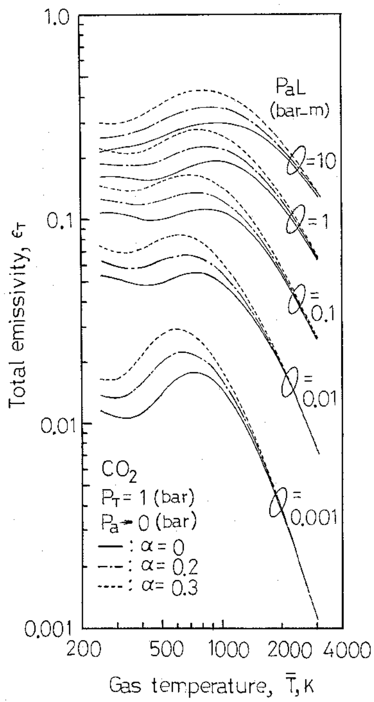

Fig. 1 Total emissivity of temperature-fluctuating carbon dioxide at a total pressure of 1 bar and 0 partial pressure.

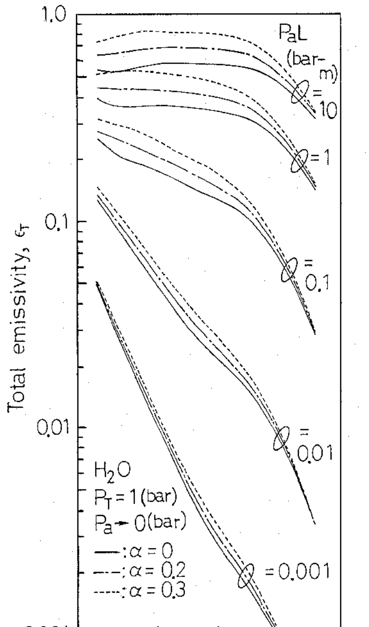

Fig. 2 Total emissivity of temperature-fluctuating water vapor at a total pressure of 1 bar and 0 partial pressure.

The results presented here are fully consistent with the previously reported fact[2–4] that even when an appreciable increase in spectral radiant intensity that is attributable to the effect of temperature fluctuations within a turbulent flame is observed in several wavelength regions, the total radiative heat fluxes from the flame are less influenced by this effect and thus can be predicted based on mean scalar fields with an acceptable accuracy. However, it should be noted that, in practical high-temperature applications where the large pressure variance can occur together with the temperature fluctuation, the total emis-